

## False Vacuum Black Holes and Universes

R. G. Daghigh and J. I. Kapusta

*School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455*

Y. Hosotani

*Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*

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We construct a black hole whose interior is the false vacuum and whose exterior is the true vacuum of a classical field theory. From the outside the metric is the usual Schwarzschild one, but from the inside the space is de Sitter with a cosmological constant determined by the energy of the false vacuum. The parameters of the field potential may allow for the false vacuum to exist for more than the present age of the universe. A potentially relevant effective field theory within the context of QCD results in a Schwarzschild radius of about 200 km.

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Normally one thinks of black holes as being created in the collapse of a star which was originally 10 to 20 times the solar mass. It is also believed that there are huge black holes of about 1 million times the solar mass near the center of galaxies. Another possibility are primordial black holes, which were created in the early universe, which presently have a mass less than 1% that of the earth, and are microscopic in dimension. One does not usually ask what goes on inside a black hole because there is an event horizon which prevents information from leaving it. In this paper we study a black hole whose interior is completely filled by the false vacuum of a classical field theory.

The type of potential we have in mind is drawn in Figure 1. For the present purpose it may conveniently be represented as a fourth-order polynomial in the real classical field  $\phi$ :

$$V(\phi) = \frac{\lambda}{4}(\phi - f_+)^2 \times \left\{ \phi^3 + \frac{4f_- - f_+}{3}\phi^2 - \frac{2f_- + f_+}{3}f_+(\phi + f_+) \right\} \\ V'(\phi) = \lambda\phi(\phi + f_-)(\phi - f_+). \quad (1)$$

Here  $f_+ > f_- > 0$ . Defining  $f = (f_+ + f_-)/2$  and  $\Delta f = f_+ - f_-$  the false vacuum at  $\phi = -f_-$  has energy density  $\epsilon_- = V(-f_-) = \frac{2}{3}\lambda f^3 \Delta f$  while the true vacuum at  $\phi = f_+$  has zero energy density.

We look for time independent, spherically symmetric solutions of Einstein's and Lagrange's equations with the metric

$$d\tau^2 = \frac{H(r)}{p^2(r)}dt^2 - \frac{dr^2}{H(r)} - r^2 d\Omega^2. \quad (2)$$

The action is

$$I = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. \quad (3)$$

Defining  $H(r) = 1 - 2GM(r)/r$ , the equations of motion are

$$\frac{1}{p} \frac{dp}{dr} = -4\pi G r \left( \frac{d\phi}{dr} \right)^2 \quad (4)$$

$$\frac{dM}{dr} = 4\pi r^2 \left[ \frac{1}{2} H \left( \frac{d\phi}{dr} \right)^2 + V(\phi) \right] \quad (5)$$

$$\frac{p}{r^2} \frac{d}{dr} \left( r^2 \frac{H}{p} \frac{d\phi}{dr} \right) = V'(\phi). \quad (6)$$

A nontrivial solution is  $\phi = -f_-$  for  $r < r_0$  and  $\phi = f_+$  for  $r > r_0$  where  $r_0$  is arbitrary. The discontinuity in the field at  $r_0$  will be analyzed below for the case of most interest to us.

For the above solution the metric functions are given by

$$p = p_{\text{in}} = \text{constant} \\ H = 1 - \frac{r^2}{r_c^2} \quad (7)$$

when  $r < r_0$  and

$$p = p_{\text{out}} = \text{constant} \\ H = 1 - \frac{r_0^3}{r_c^2 r} \quad (8)$$

when  $r > r_0$ . The two constants  $p_{\text{in}}$  and  $p_{\text{out}}$  are not the same; generally one of them may be set to one by scaling the time variable. The  $r_c$  is a critical radius related to a cosmological constant  $\Lambda = 3/r_c^2$  and given by

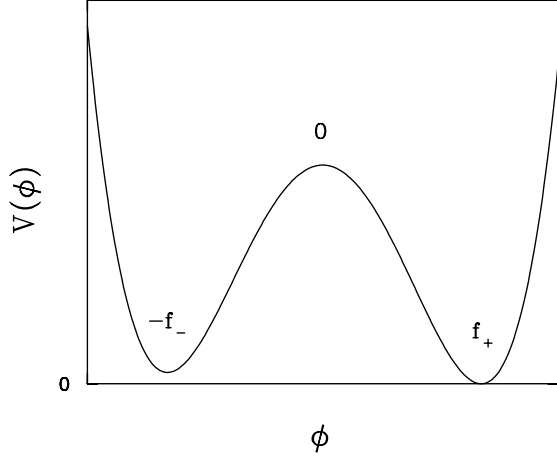


FIG. 1. An example of a scalar potential with one local minimum (false vacuum) and one global minimum (true minimum). The potential drawn is a fourth-order polynomial.

$$r_c = \sqrt{\frac{3}{8\pi G\epsilon_-}} = \frac{3m_P}{4f\sqrt{\pi\lambda f\Delta f}} \quad (9)$$

Here  $m_P = G^{-1/2} = 1.22 \times 10^{28}$  eV is the Planck mass. The function  $H(r)$  is plotted in Figure 2. It has a cusp at the edge of the bubble,  $r = r_0$ . For  $r_0 < r_c$  we have a false vacuum bubble, which may be unstable, in a space with zero cosmological constant and with the gravitational field taken into account. When  $r_0 = r_c$  there is a single horizon. From the outside it appears as a black hole with Schwarzschild radius  $r_S = r_c$ . The outside metric is just the usual Schwarzschild one as guaranteed by the Birkhoff Theorem. When  $r_0 > r_c$ ,  $H$  crosses zero twice and there are two horizons, an inner one at  $r_c$  and an outer one at  $r_S = r_0^3/r_c^2 > r_c$ . Anyone on the outside would only see the Schwarzschild solution, a black hole, and would have no idea what is inside. For this last case  $g_{tt} < 0$  and  $g_{rr} > 0$  in the shell  $r_c < r < r_S$ , indicating that the time and radial coordinates have switched roles. In this shell one can make a change of coordinates, redefining time, after which the solution we have found is no longer time independent in the new variables.

The case of interest in this paper is when  $r_0 = r_c$ , that is, a black hole with false vacuum on the inside and true vacuum on the outside. The fact that no information can cross the boundary is recognized in two different manners. For an outside observer no signal from inside can cross the event horizon of the Schwarzschild black hole. For an inside observer any signal emitted outwards can reach at most the cosmological horizon as the de Sitter space expands. Though completely different, these describe the same physical event.

The first task is to study the behavior of  $\phi$  near the surface. Is the jump in  $\phi$  really an acceptable or approximate solution? Let us place the surface at the local maximum of the potential and linearize the equation of

motion for  $\phi$  around  $\phi = 0$ . Then  $\phi$  would be expected to roll down to  $-f_-$  on the inside and to  $f_+$  on the outside. First we take a perturbative approach with  $H(r)$  as a given background field, the same as above. Defining  $x = r - r_c$ , and considering  $|x| \ll r_c$ , the equation just inside ( $x < 0$ ) is

$$x \frac{d^2 \phi}{dx^2} + \frac{d\phi}{dx} - \frac{1}{2l} \phi = 0 \quad (10)$$

and just outside ( $x > 0$ ) is

$$x \frac{d^2 \phi}{dx^2} + \frac{d\phi}{dx} + \frac{1}{l} \phi = 0. \quad (11)$$

The length scale,  $l = 1/\lambda f_+ f_- r_c$ , is very small compared to  $r_c$  unless  $f$  is comparable to the Planck mass. The solutions to these equations are

$$\begin{aligned} \phi(x < 0) &= a_1 J_0(\sqrt{-2x/l}) + a_2 N_0(\sqrt{-2x/l}) \\ \phi(x > 0) &= b_1 J_0(2\sqrt{x/l}) + b_2 N_0(2\sqrt{x/l}). \end{aligned} \quad (12)$$

If  $p(x)$  were constant in the entire region, then the condition that  $\phi(x = 0) = 0$  would mean that all coefficients  $a_1, a_2, b_1, b_2$  are zero. With the variation in  $p(x)$  ignored, there is no smooth transition from  $-f_-$  on the inside to  $f_+$  on the outside, at least not in this perturbative approach. There must be singular behavior at  $x = 0$ . After all, there is a horizon separating the outside from the inside.

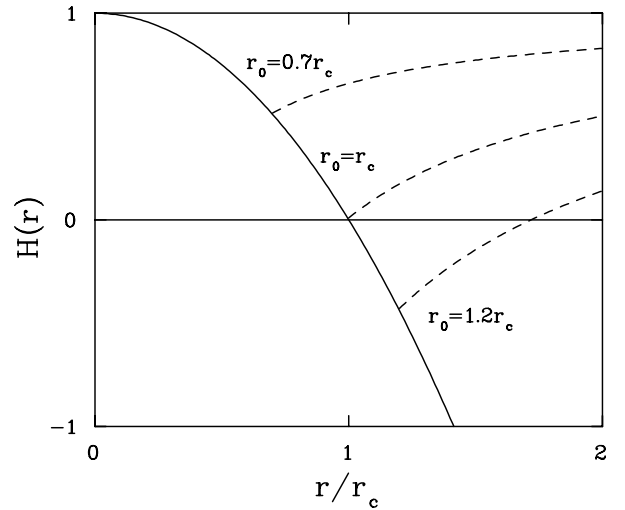


FIG. 2. The metric function  $H(r)$  resulting from the classical field configuration  $\phi(r) = -f_- \theta(r_0 - r) + f_+ \theta(r - r_0)$ . There is a horizon at every point where  $H = 0$ .  $H(r)$  is given by the solid line for  $r < r_0$  and by the dashed line for  $r > r_0$ .

There is a simple way to understand this behavior. Eq. (6) in the surface region can be obtained by minimizing the expression

$$\tilde{I} = \int_{\text{surface}} dx \frac{1}{p(x)} \times \left\{ \frac{1}{2} H(x) \left( \frac{d\phi}{dx} \right)^2 + V(\phi(x)) \right\}. \quad (13)$$

Let  $w$  characterize the width of the surface. In order of magnitude we can replace  $d\phi/dx$  with  $2f/w$ ,  $V(\phi(x))$  with  $\lambda f^4/4$ , and  $\int dx$  with  $w$ . In the absence of gravity,  $H = p = 1$ ,  $\tilde{I}$  as a function of its width is approximately

$$\frac{f^2}{w} + \lambda f^4 w.$$

This is minimized when  $w^2 \sim 1/\lambda f^2$ , meaning that  $w$  is approximately equal to the correlation length, the usual result. To include gravity we replace  $H(x)$  with  $w/r_c$ . ( $H$  vanishes linearly at  $r = r_c$  with negative slope on the inside and positive slope on the outside.) If  $p(x) = 1$ ,  $\tilde{I}$  as a function of  $w$  is approximately

$$\frac{f^2}{r_c} + \lambda f^4 w.$$

This is minimized for  $w = 0$ , forcing a jump in the field.

The function  $p(x)$ , however, varies from  $p_{\text{in}}$  to  $p_{\text{out}}$  at the surface as Eq. (4) dictates. Taking  $\phi = 2fx/w$  in the surface region  $-w/2 < x < w/2$ , we have  $p(x) = \exp(-\alpha r_c x/w^2)$  where  $\alpha = 16\pi G f^2 = 16\pi(f/m_P)^2$ . As  $w \rightarrow 0$ ,  $p_{\text{out}} \rightarrow 0$  so that  $\tilde{I}$  diverges. Approximating  $H(x)$  by  $-2x/r_c$  ( $x < 0$ ) and  $x/r_c$  ( $x > 0$ ) in the surface region,  $\tilde{I}$  is given in terms of  $y = w/(\alpha r_c)$  by

$$\tilde{I} = \alpha r_c \lambda f^4 F(y), \quad F(y) = \gamma [g_1(y) + 2g_1(-y)] + [g_2(y) - g_2(-y)], \quad (14)$$

where  $\gamma = 2\Delta f/9f$ ,  $g_1(y) = y\{y + (\frac{1}{2} - y)e^{1/2y}\}$ , and  $g_2(y) = 8y^4(12y^2 - 6y + 1)e^{1/2y}$ . The  $g_1$  and  $g_2$  represent contributions from the kinetic and potential terms, respectively. The constant  $\alpha$  is tiny; it is  $\sim 10^{-38}$  for  $f = 100$  MeV.  $\gamma$  is smaller than  $10^{-2}$ .  $F(y) \sim y/6$  for  $y \gg 1$ , whereas  $F(y) \sim \frac{1}{2}\gamma y e^{1/2y}$  for  $y \ll 1$ . For  $\gamma < 10^{-2}$  the  $g_2$ -terms in  $F(y)$  dominate except for  $y \ll 1$ .  $\tilde{I}$  is minimized at  $y \sim 0.18$  almost independent of the value of  $\gamma$  ( $< 10^{-2}$ ), or at  $w \sim 4(f/\lambda\Delta f)^{1/2} \cdot l_P$  where  $l_P = 1/m_P = 1.6 \times 10^{-35}$  m is the Planck length. The surface is very thin compared with the size of the bubble. Simple calculations show that the surface energy is smaller than the volume energy by a factor of  $f^3/(m_P^2 \Delta f)$ .

The lifetime of the false vacuum may be determined semiclassically using the methods of Coleman *et al.* without [1] or with [2] gravity taken into account. The rate per unit volume for making a transition from the false vacuum to the true vacuum is expressed as

$$\frac{\Gamma}{V} = A e^{-B/\hbar} [1 + \mathcal{O}(\hbar)] \quad (15)$$

where Planck's constant has been used here to emphasize the semiclassical nature of the tunneling rate. For the potential being used in this paper we find the  $O(4)$ , Euclidean space, bounce action (neglecting gravity) to be

$$B_0 = \frac{36\pi^2}{\lambda} \left( \frac{f}{\Delta f} \right)^3 \quad (16)$$

where the radius of the critical size bubble which nucleates the transition is

$$\rho_c = \frac{3}{\Delta f} \sqrt{\frac{2}{\lambda}}. \quad (17)$$

For any sensible estimate of the coefficient  $A$  the lifetime of the false vacuum will exceed the present age of the universe when the condition

$$\lambda \left( \frac{\Delta f}{f} \right)^3 < 1 \quad (18)$$

is satisfied. This calculation is based on the thin wall approximation, which is valid when the critical radius is large compared to the coherence length of the potential, namely  $1/\sqrt{|V''|}$ . This condition translates into  $\Delta f \ll 6f$ . With gravity included the bounce action is

$$B = \frac{B_0}{[1 + (\rho_c/2r_c)^2]^2}. \quad (19)$$

Gravitational effects are negligible when  $\rho_c \ll r_c$ , or  $f\sqrt{f/\Delta f} \ll m_P$ . When  $\rho_c > r_c$ , the black hole is too small to accommodate even a single nucleation bubble, or in other words the bounce solution does not have  $O(4)$  symmetry. Nucleation of a bubble is further suppressed.

The proper time for a light signal starting a distance  $\delta \ll r_c$  from the event horizon to propagate to the origin, as computed with the de Sitter metric, is  $\Delta t = \frac{1}{2} r_c \ln(2r_c/\delta)$ . For  $\delta$  of order of the surface thickness this is approximately  $r_c \ln(m_P/f)$ , which is much bigger than  $r_c$  for  $f \ll m_P$ . This is the minimum time for a symmetry restoring signal, originating near the surface, to destroy the bubble.

An outside observer would have no information on the state of the universe inside the Schwarzschild radius. Such an observer would see a black hole with mass  $M_c = m_P^2 r_c/2$ . An observer on the inside, however, would be living in a de Sitter space with a cosmological constant  $\Lambda = 3/r_c^2$ . A standard change of coordinates [3] puts the metric on the inside in the form

$$dr^2 = dt'^2 - e^{2t'/r_c} (dr'^2 + r'^2 d\Omega'^2). \quad (20)$$

From these considerations we may identify a Hubble constant  $= 1/r_c$ .

To make matters interesting, let us suppose that the cosmological constant suggested by recent observations of distant Type Ia supernovae [4] arises from the universe actually being in a false vacuum state. A best

fit to all cosmological data [5] reveals that the present energy density of the universe has the critical value of  $\epsilon_c = 3H_0^2/8\pi G$ , with one-third of it consisting of ordinary matter and two-thirds of it contributed by the cosmological constant. With a present value of the Hubble constant of  $H_0 = 65 \text{ km/s-Mpc}$  we find that

$$r_c = \sqrt{\frac{3}{2}} \frac{1}{H_0} = 1.7 \times 10^{26} \text{ m} = 5.5 \text{ Gpc} \quad (21)$$

and

$$(\lambda f^3 \Delta f)^{1/4} = \epsilon_c^{1/4} = 2.4 \times 10^{-3} \text{ eV}. \quad (22)$$

This constraint on the parameters of the potential,  $\lambda$ ,  $f$  and  $\Delta f$ , is entirely consistent with the constraints imposed by the lifetime of the false vacuum exceeding the present age of the universe.

The above discussion is pure speculation of course. A cosmological constant, if it exists, may have its origins elsewhere. But if it does arise from a false vacuum, a variety of questions immediately present themselves. Is  $\phi$  a new field, not present in the standard model of particle physics, whose only purpose is this? Where does the energy scale of 2.4 meV come from? Why should  $V(\phi)$  have a global minimum of 0, especially when quantum mechanical fluctuations are taken into account? To these questions we have no answers.

Another amusing possibility is that black holes believed to exist near the center of galaxies are false vacuum black holes. The mass of the critical false vacuum black hole is given by

$$M_c = \sqrt{\frac{3}{32\pi G^3 \epsilon_-}} = \frac{0.282 \cdot M_{\text{Sun}}}{\sqrt{\epsilon_-/\text{GeV}^4}} \quad (23)$$

where  $M_{\text{Sun}}$  is the solar mass. In other words the scale  $\epsilon_-^{1/4} = 1 \text{ MeV}$  corresponds to a black hole of 1 million solar mass.

Yet another amusing possibility is an analogy to spin glasses [6] and, possibly, QCD. A spin glass is characterized by a phase space that has a complicated landscape of troughs and valleys. At low temperatures barriers become very large, and the system may become trapped in a metastable state for the whole duration of an experiment. The possibility of a second minimum in the effective potential has been noted in regards to the chiral phase transition in QCD [7]. Chiral symmetry is explicitly broken by the nonzero masses of the up and down quarks. In the context of a low energy representation of QCD by the linear sigma model, it was argued in [7] that one should consider the most general symmetry breaking potential which is at most fourth order in the fields.

$$V_{SB} = - \sum_{n=1}^4 \frac{\delta_n}{n!} \sigma^n + (\delta_5 \sigma + \delta_6 \sigma^2) \pi^2. \quad (24)$$

(Other symmetry-breaking terms one might think of adding simply amount to a redefinition of the eight parameters  $\lambda, f, \delta_n$ .) Historically the most used potential is  $-f_\pi m_\pi^2 \sigma$ . The full potential then has a global minimum at  $\sigma = f_\pi$  and a saddle point near  $\sigma = -f_\pi$  (it is a local minimum in the  $\sigma$  direction but a local maximum in the  $\pi$  direction). Another one is  $\frac{1}{2} m_\pi^2 \pi^2$ , which has two degenerate minima at  $\sigma = \pm f_\pi$ . It is possible to choose the symmetry breaking potential so that the full potential has two nearly degenerate minima in the full  $\sigma - \pi$  space such that small field fluctuations around one or the other of them reproduces known physics. A natural estimate is  $\lambda = 10$ ,  $f = 100 \text{ MeV}$ , and  $\Delta f = 2 \text{ MeV}$ . Then  $r_c = 230 \text{ km}$ , and such a black hole would have a mass of about 80 suns. Such astronomical objects may be left over from the QCD phase transition in the early universe. One may even speculate that they formed the seeds for galactic black holes.

The most outrageous possibility is that we might be living inside an enormous black hole. If so, what is outside our universe?

## ACKNOWLEDGEMENTS

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